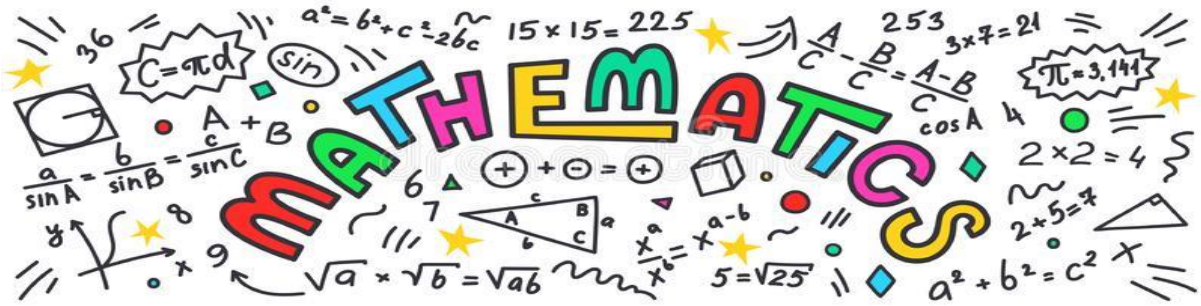


Higham Lane School

A Level Further Maths

Challenge Booklet

$$\sqrt{-1} \quad 2^3 \quad \Sigma \quad \pi$$

and it was delicious!

Reading List

As a student who is choosing to study A Level Further Maths, it's logical to assume you have an interest in the subject. The following books may provide additional reading for you beyond those suggest for Maths A Level.

How to think like a Mathematician *by Kevin Houston*

How to Solve it *by George Polya*

Fermat's Last Theorem *by Simon Singh*

The Music of Primes *by Marcus De Sautoy*

Euler's Pioneering Equation *by Robin Wilson*

Archimedes' Revenge *by P.Hoffman*

Chaos and Fractals, An elemental Introduction *by David P. Feldman*

Algorithms Unlocked *by Thomas H. Cormen*

The Tiger That Isn't: Seeing Through a World of Numbers *by Andrew Dilnot and Michael Blastland*

Why Do Buses Come in Threes?: The Hidden Mathematics of Everyday Life

By Rob Eastaway & Jeremy Wyndham

There is also much more digital media out there from videos, blogs, podcasts and interactive website. Here are a few you might want to visit.

Websites

http://vihart.com/

http://dailydesmos.com/

http://www.estimation180.com/

http://www.visualpatterns.org/

https://allthenewsthatsfittomath.blogspot.com/

http://ptolemy.co.uk/2007/09/19/primitives/

https://www.ilovemathsgames.com/index.html

https://www.transum.org/

https://www.mathsisfun.com/data/quincunx.html

Graphing Software

If you haven't already been using online graphical software now is the time to familiarise yourself with it, we recommend either Desmos or Geogebra both free software and easily downloadable. Maybe have a look at a few online tutorials there are plenty of videos out there.

https://www.desmos.com/calculator

https://www.geogebra.org/

Activities

The following tasks are provided for you to think more in depth about the Maths you have already met at GCSE Maths. Most have a link to a website where you can seek further guidance if needed.

TASK 1

https://undergroundmathematics.org/quadratics/inequalities-for-some-occasions

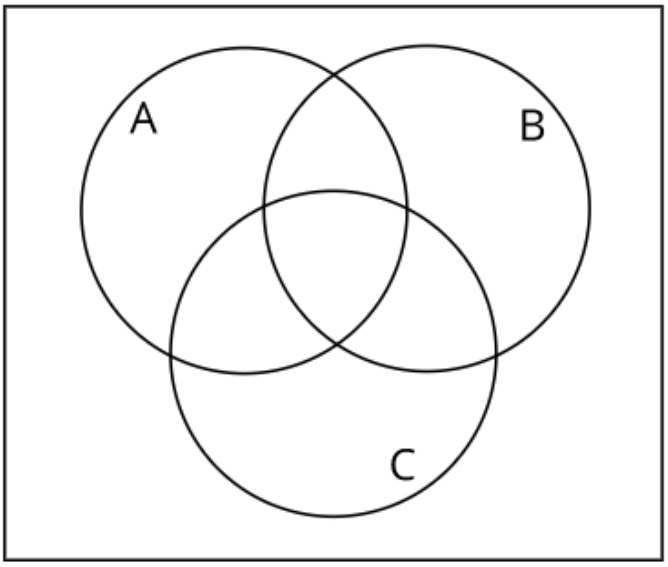
Can we find a quadratic inequality for each region on the Venn diagram?

The regions are defined as follows.

A: The solution set is a subset of $x \leq 1$.

B: The solutions are given by $a \leq x \leq b$ where a and b are real numbers.

C: The inequality is satisfied by $x = 4$, e.g. $x = 4$ satisfies the inequality $x \geq 2$.

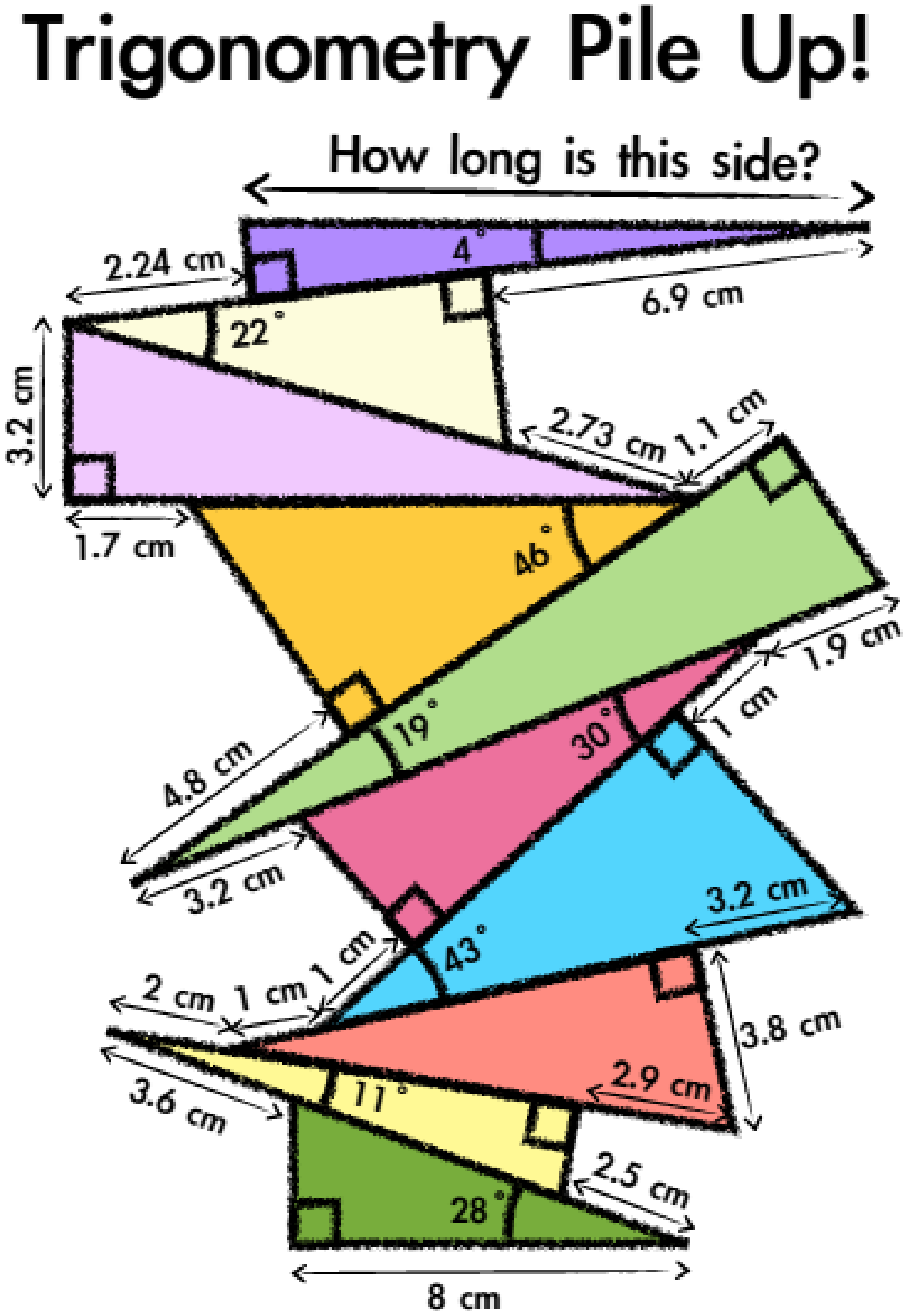


Here are some possible inequalities. Start by placing these into the correct region of the Venn diagram.

① $x^2 \leq 9$	② $11x \geq 2x^2$	③ $x^2 + 3 \geq 2$	④ $3x^2 \geq 21x - 30$
⑤ $x^2 \leq -x$	⑥ $x^2 \leq x - 2$	⑦ $6x^2 - 1 \geq 5x$	⑧ $-2x^2 \leq x - 6$

TASK 2

http://www.greatmathsteachingideas.com/2012/03/12/trigonometry-pile-up/



TASK 3

https://nrich.maths.org/5970

Simultaneous Equations Sudoku

c		m				h			19
	f						e		10
			m						
			k		g	m	c		23
g				p					11
							h	f	14
	g				m				13
	a			e			k	h	11
k			c		f	a			22
17	19		16	5	22	14	16	14	

Rules of Equation Sudoku

Like the standard sudoku, this sudoku variant has two basic rules:

1. Each column, each row and each box (3x3 subgrid) must have the numbers 1 through 9.
2. No column, row or box can have two squares with the same number.
3. The puzzle can be solved by finding the values of the 9 given variables in the squares of the 9x9 grid. At the bottom and right side of the 9x9 grid are numbers, each of which is the sum of a column or row of variables. Altogether a set of 16 equations can be formed from the columns and rows of variables and constants.

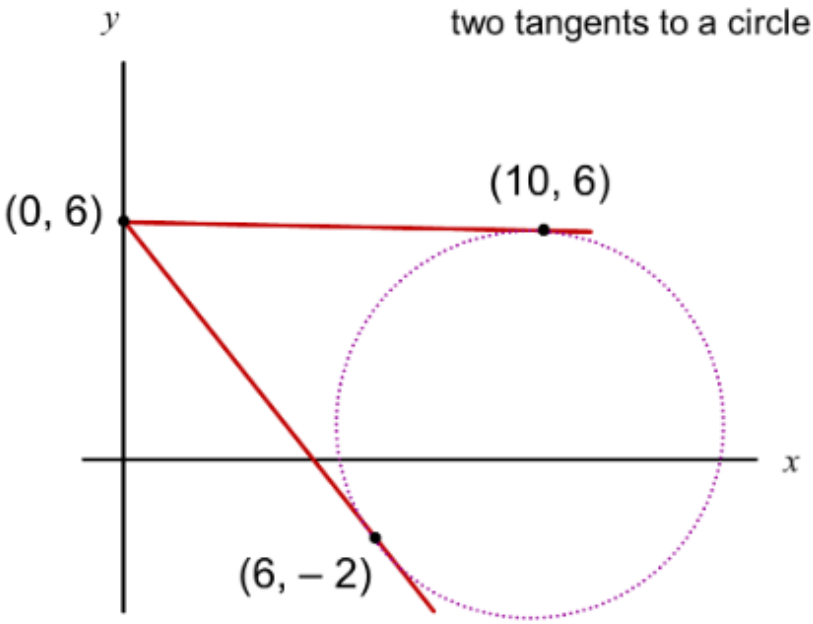
Task 4

What's the Difference?

$2x^2 = 18$	$(2x)^2 = 18$	$2x^2 + 1 = 18$	$(2x + 1)^2 = 18$	$2(x + 1)^2 = 18$

Task 5

https://donsteward.blogspot.com/2017/11/two-tangents-meet-on-y-axis.html



two tangents to a circle meet at (0, 6)

what are the coordinates of the centre of the circle?

Two tangents to a circle

More on circles here

https://www.transum.org/Maths/Exercise/Circle_Equations.asp

Task 6

https://www.openmiddle.com/perfect-squares/

PERFECT SQUARES

Directions: Using the digits 1- 9, at most one time each, to fill in the boxes to make each expression evaluate to a perfect square number.

Extension/Challenge: What is the largest/smallest square number you can make? How many different perfect square numbers could be made?

$18 \times \square \times 2$

$\square \times 14 \times \square$

$\square \times 15 \times 3$

$2 \times \square$

$6 \times \square \times 2 \times \square$

Task 7

https://undergroundmathematics.org/thinking-about-numbers/absurd

Ab-surd!

Each line is a set of equivalent fractions. Fill in the blanks in the fractions to make each line complete, including the multiplier used to get from one fraction to the next.

(1) $\frac{1}{\sqrt{2}}\left(\times\ \_\_\_\right) = \frac{\sqrt{2}}{\ \ }\left(\times\ \_\_\_\right) = \frac{\sqrt{6}}{\ \ }\left(\times\ \_\_\_\right) = \frac{\ \ }{6}$

(2) $\frac{2}{5\sqrt{3}}\left(\times\ \_\_\_\right) = \frac{\ \ }{15}\left(\times\ \_\_\_\right) = \frac{2\sqrt{6}}{\ \ }\left(\times\ \_\_\_\right) = \frac{\ \ }{60}$

(3) $\frac{5}{2+\sqrt{2}}\left(\times\ \_\_\_\right) = \frac{10-5\sqrt{2}}{\ \ }\left(\times\ \_\_\_\right) = \frac{\ \ }{20+10\sqrt{2}}$

(4) $\frac{2-\sqrt{3}}{4}\left(\times\ \_\_\_\right) = \frac{\ \ }{8+4\sqrt{3}}\left(\times\ \_\_\_\right) = \frac{\ \ }{16}$

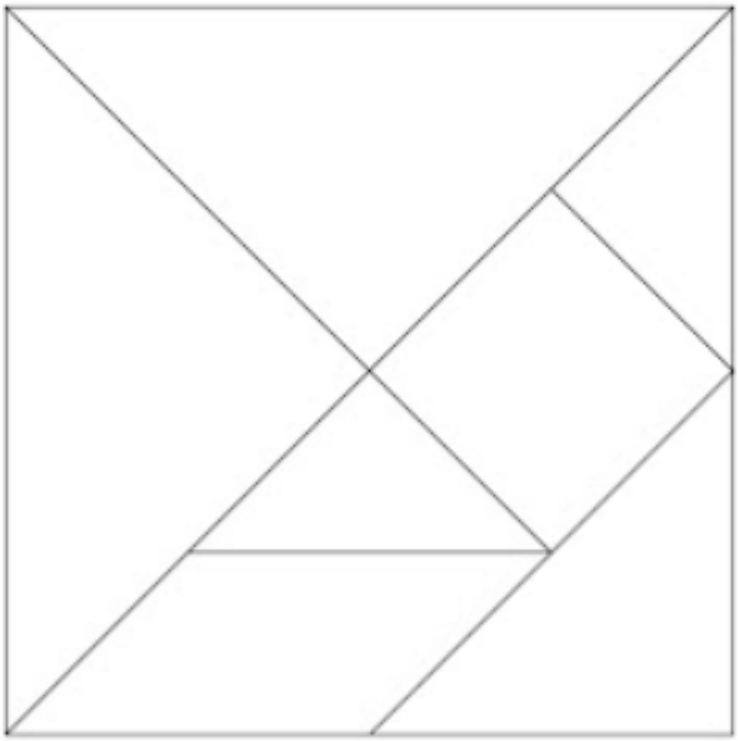
✱ A *rationalised fraction* is one whose denominator is a whole number. These are usually easier to work with than fractions with square roots in their denominators.

- Identify the rationalised fractions in the above lines. What do you notice about the multipliers when moving from a fraction with a surd (square root) in the denominator to a rationalised fraction?
- How would you rationalise fractions in the following form: $\frac{a}{\sqrt{b}}$, $\frac{a}{b\sqrt{c}}$ and $\frac{a}{b+\sqrt{c}}$?
- Is there more than one way to rationalise a fraction?

Task 8

https://donsteward.blogspot.com/search?q=tangram+vectors

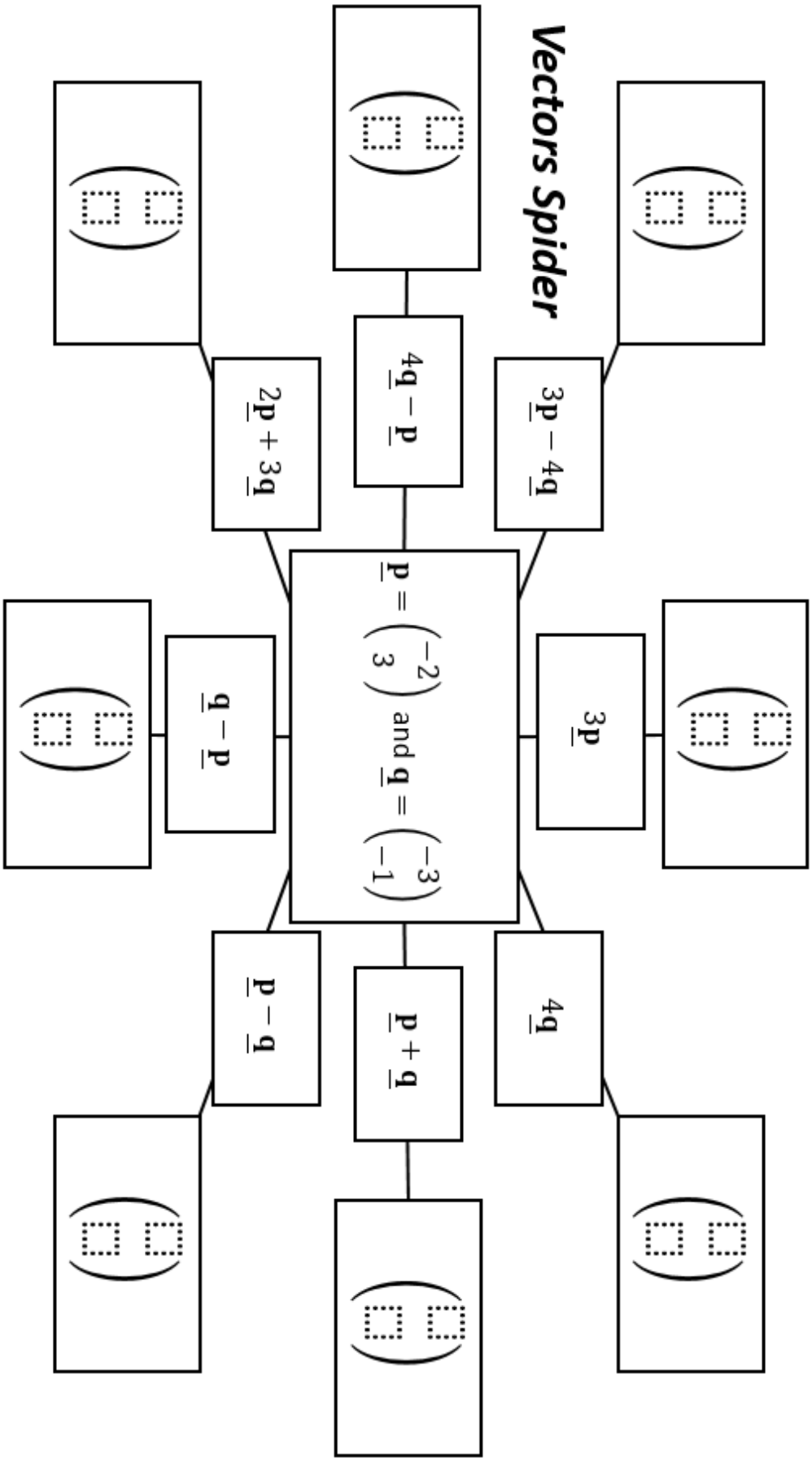
Tangram Vectors



given that the parallelogram has sides with vectors **a** and **b** heading away from the bottom left hand corner

what are the vectors for other lines in the tangram puzzle?

Task 9


Vectors Spider
$\underline{\mathbf{p}} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\underline{\mathbf{q}} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$
$3\underline{\mathbf{p}} - 4\underline{\mathbf{q}}$
$4\underline{\mathbf{q}} - \underline{\mathbf{p}}$
$2\underline{\mathbf{p}} + 3\underline{\mathbf{q}}$
$3\underline{\mathbf{p}}$
$\underline{\mathbf{q}} - \underline{\mathbf{p}}$
$\underline{\mathbf{p}} - \underline{\mathbf{q}}$
$\underline{\mathbf{p}} + \underline{\mathbf{q}}$
$4\underline{\mathbf{q}}$

Task 10

https://undergroundmathematics.org/geometry-of-equations/r5281

Can we find an integer solution to three simultaneous inequalities?

From the inequalities

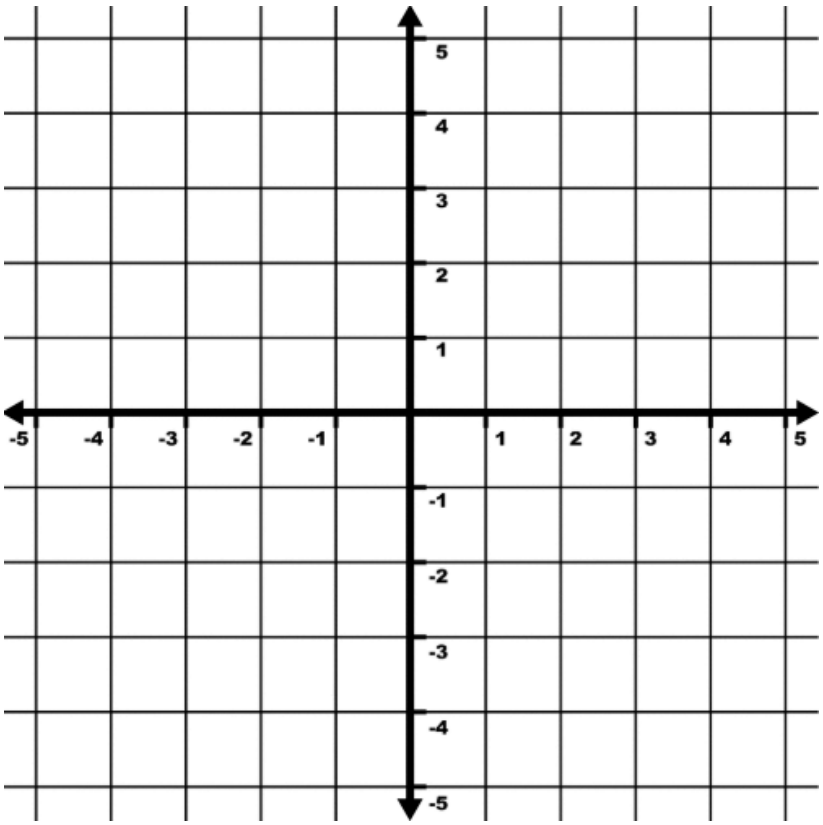
$$y-2x>0, \qquad x+y>3, \qquad 2y-x<5$$

deduce that

$$\frac{1}{3}<x<\frac{5}{3}, \qquad 2<y<\frac{10}{3},$$

and hence that the given inequalities cannot be satisfied simultaneously by integral values of x and y.

[*The phrase 'integral values' means the same as 'integer values'.*]



Task 11

Indices – follow me…….

START	$\left(\frac{7}{2}\right)$	4	0.027	$\frac{16}{9}$	$\frac{2}{5}$	$-\frac{1}{8}$	$\frac{1}{400}$	1	27
A	T	P	N	1	S	T	R	H	F
8^{-1}	$\left(\frac{1}{7}\right)^{-2}$	$4^{\frac{3}{2}}$	$\left(\frac{1}{3}\right)^{-1}$	FINISH	$\left(\frac{27}{64}\right)^{-\frac{2}{3}}$	$(-3)^{-2}$	$\left(\frac{5}{3}\right)^{-2}$	$\left(\frac{9}{5}\right)^{-1}$	$36^{\frac{3}{2}}$

16	8	$\frac{1}{8}$	$\frac{9}{25}$	216	$\frac{1}{64}$	64	1000	$\frac{8}{3}$	$\frac{1}{36}$
I	O	N	O	Z	Y	T	O	I	E
$(0.3)^3$	$27^{\frac{2}{3}}$	8^{-2}	$\left(\frac{9}{64}\right)^{-\frac{1}{2}}$	6^{-2}	$(-2)^{-3}$	$\left(\frac{5}{3}\right)^{0}$	$81^{\frac{3}{4}}$	$\left(\frac{25}{4}\right)^{-\frac{1}{2}}$	20^{-2}

125	3	9	49	$\frac{1}{9}$	$\left(\frac{5}{9}\right)$	$\left(\frac{1}{1000}\right)$
E	G	W	O	H	E	R
$100^{-\frac{3}{2}}$	$\left(\frac{2}{7}\right)^{-1}$	$25^{\frac{3}{2}}$	$\left(\frac{1}{4}\right)^{-3}$	$(0.25)^{-2}$	$8^{\frac{2}{3}}$	$100^{\frac{3}{2}}$

Task 12

Races

Five children (Ahmed, Bachendri, Charlie, Daniel and Emily) raced each other. First they raced to the spreading chestnut tree, and then they raced back to their starting point. The following facts are known:

(i) Ahmed was fourth in the race to the tree.

(ii) The person who was last to the tree managed to win the race back.

(iii) The person who won the race to the tree was third on the way back.

(iv) The person who was third in the race to the tree was second on the way back.

(v) Bachendri was fourth on the way back.

(vi) Charlie reached the tree before Daniel.

(vii) Charlie got back to the start before Emily.

For each race (to the tree and back again), write down the order in which the children finished.